

10. Analyzing of the lower sector of the Danube evolution based on logistic map

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Abstract: This study aims to use the logistic map in order to determine the speed of evolution of the various components of the lower Danube sector. Due to the ability of logistic map to capture the speed of evolution of some processes, and the degree of self-similarity of the data, a similar study was an older desire of ours. As a large area of land generally remains uncovered by studies and it is difficult to obtain data collected at the same time, all this leads to a degree of inconsistency in the data obtained. The analysis based on these data sets revealed on the one hand how the evolution speed of the lower sector of the Danube changed, on the one hand and the search oscillations, on the distribution in space of the human settlements on the other hand. Within the lower Danube sector, 8 clusters were identified based on the spatial distribution. All these played a feed-back role. Since the Neolithic Period, small islands appeared in the area between Dinogetia and Aegyssus Promotory. However, the presence of human settlements in the area of the Letea sandbank indicates the presence of fresh water, wood (for construction and fire) and access to the sea, so it is very possible that the Letea and Caraorman sandbanks were occupied of forests in the Byzantine Period. Basis of the distribution of human settlements, the evolution of Danube lower sector can be described. Within this paper, the "behavior" of the Danube, in the area of its lower sector, was analyzed on the basis of the logistic map of the human settlements distribution.

Keywords: Lower Danube, promontories, cluster, logistic map, historical periods.

INTRODUCTION

This paper represents an older desire of us to analyse the data according to their logistic map. The logistic map is a polynomial mapping (equivalently, recurrence relation) of degree 2, often cited as an archetypal example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations.

In his 1926 doctoral thesis, Cauer was already sketching a complete program for network synthesis as a solution to the inverse problem of circuit analysis: Given the external behaviour of a linear passive one-port in terms of a driving-point impedance as a prescribed function of frequency, how does one find internally passive realizations for this 'black-box'? He then shows that the synthesis problem requires the systematic solution of three main issues concerning the realizability, approximation, and realization (Cauer, 2000).

On the basis of this fundamentally new concept of external equivalence of passive linear networks under transformations of internal variables, Cauer was able to state the problem of linear circuit synthesis (Cauer, 2000).

Systems as diverse as genetic networks and human settlements or the World Wide Web are best described as networks with complex topology. A common property of many large networks is that the vertex connectivity follows a scale-free power-law distribution. This feature was found to be a consequence of two generic mechanisms: (i) networks expand continuously by the addition of new vertices, and (ii) new vertices attach preferentially to sites that are already well connected. A model based on these two ingredients reproduces the observed stationary scale-free distributions, which indicates that the development of large networks is governed by robust self-organizing phenomena that go beyond the particulars of the individual systems. (Barabasi & Reka, 1999)

Networks have in recent years emerged as an invaluable tool for describing and quantifying complex systems in many branches of science. Recent studies suggest that networks often exhibit hierarchical organization, where vertices divide into groups that further subdivide into groups of groups, and so forth over multiple scales. In many cases these groups are found to correspond to known functional units,

such as ecological niches in food webs, modules in biochemical networks (protein interaction networks, metabolic networks, or genetic regulatory networks), or communities in social networks. (Clauset & Newman, 2008)

Networks of coupled dynamical systems have been used to model human settlements and many other self-organizing systems. Ordinarily, the connection topology is assumed to be either completely regular or completely random. But many biological, technological and social networks lie somewhere between these two extremes. Here we explore simple models of networks that can be tuned through this middle ground: regular networks 'rewired' to introduce increasing amounts of disorder. We find that these systems can be highly clustered, like regular lattices, yet have small characteristic path lengths, like random graphs. (Duncan & Strogatz, 1998)

Networks, also called graphs by mathematicians, provide a useful abstraction of the structure of many complex systems, ranging from social systems and computer networks to biological networks and the state spaces of physical systems. In the past decade there have been significant advances in experiments to determine the topological structure of networked systems, but there remain substantial challenges in extracting scientific understanding from the large quantities of data produced by the experiments. A variety of basic measures and metrics are available that can tell us about small-scale structure in networks, such as correlations, connections and recurrent patterns, but it is considerably more difficult to quantify structure on medium and large scales, to understand the 'big picture'. (Newman, 2012)

One of the most relevant features of graphs representing real systems is community structure, or clustering, i.e. the organization of vertices in clusters, with many edges joining vertices of the same cluster and comparatively few edges joining vertices of different clusters. Such clusters, or communities, can be considered as fairly independent compartments of a graph, playing a similar role like, e.g., the tissues or the organs in the human body. Detecting communities is of great importance in sociology, biology and computer science, disciplines where systems are often represented as graphs. (Santo, 2010)

Thus, this approach offers a better approximation of a black box model, because it uses the same data type. In this case, the distribution of human settlements (as a network) by periods is used. Two sets of data were used for the analysis. The set of the early years of a historical period, and the end years is used.

For data analysis, entropy (the logistic distribution entropy) was used, as a measure of the degree of "data roughness". The obtained values were introduced in the calculation of the logistic map, together with the values of the cumulative distribution function of the logistic distribution.

The analysis based on these data sets revealed on the one hand how the evolution speed of the Danube Delta and lower sector of the Danube changed, on the one other hand and the search oscillations, on the distribution in space of the human settlements.

All data used in this study were georeferenced in the Stereo 70 System, on the Krasovsky 1940 ellipsoid and the Black Sea 1975 altimetric reference system.

MATERIALS AND METHODS

As material used, I have to mention the map of the Danube from its source to the flow into the Black sea prepared by Nicolas Sanson after V. Coronelli. Large title cartouche lower left with dedication to M. Dauphin. Covers the length of the Danube from its source -west of Ulm to the Black Sea. The elaborate title cartouche has portraits of local rulers, figures symbolizing river sources, and a variety of weapons and characters including the military struggles between Europe and the Turks throughout this area. (Covents & Mortier, 2000).

The field observations made on the natural environment and archaeological sites are generally punctual and implies a degree of inconsistency in the data obtained fractal dimension was used to fill the gaps.

The first step was to calculate x_n , with the definition domain $(0, 1]$, based on the cumulative distribution function of the logistic distribution (see formula 1). The value of historical periods in years was noted with x . The x_n

The next step was to calculate the α , with the definition domain (0, 1], value by reporting the logistic distribution entropy value to 1 (see formula 2).

The last step was the calculation of x_{n+1} based on the logistic map (see formula 3). (Young , 2014)

$$x_n = \frac{1}{1 + e^{\frac{-(x-m(x))}{|\sigma_x|}}} \quad (1)$$

$$\alpha = \frac{1}{|\ln(\sigma_x) + 2|} \quad (2)$$

$$x_{n+1} = 4\alpha x_n(1 + x_n) \quad (3)$$

The **cumulative distribution function** of the logistic distribution with **blue color** and **logistic map** with **red color** help us to share the evolution of the lower Danube sector in 9 stages (see Figure 1):

1. Black Sea lake stage (10000 - 8000 years B.C.);
2. marine transgression stage (8000 - 6500 years B.C.);
3. lagoon stage (6500 - 5500 years B.C.);
4. upstream small islands formation, St. Gheorghe I and Sulina Delta downstream genesis, Danube River follow the main land shape stage (5500 - 4000 years B.C.);
5. St. Gheorghe I Delta fluvio-marine, Chilia Delta (started to evolve) and Danube River follow the main land shape (4000 - 3500 years B.C.);
6. Sulina Delta (fluvial side), Chilia Delta (fluvial side) and Danube River follow the main land shape stage (3500 - 2500 B.C.);
7. The sub-phase of the St. Gheorghe Delta (fluvio-marine side) with Delta Sulina (fluvial side and northern half of marine side), the Chilia Delta (fluvial side - half) and many upstream small islands formation stage
8. The stage of the Delta Sulina (marine side) and the Chilia Delta (fluvial side), between Dacian Period – Roman Period
9. The phase of the Delta Sulina erosion and the development of the Lagunar Complex, the Danube River begins the development of the Măcin-Grindu sector and the Somova-Parches area (Byzantine Period)

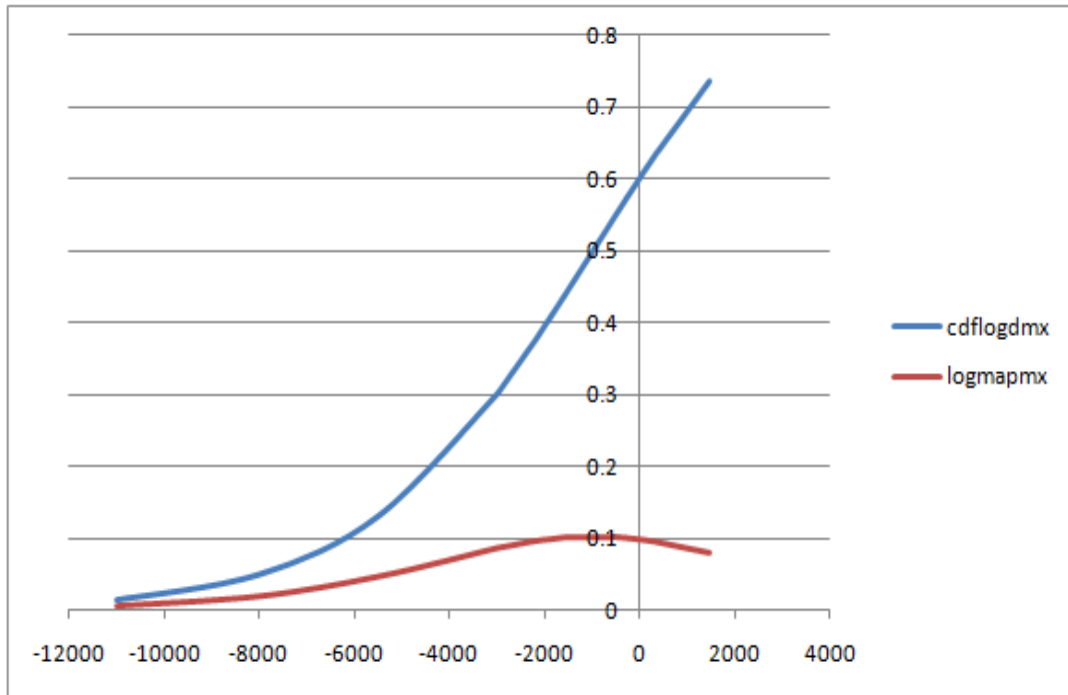


Figure 1. The cumulative distribution function of the logistic distribution with blue color and logistic map with red color

The data exposed in the Danube Delta Eco-Touristic Museum Centre were correlated with Black Sea evolution level. The evolution of the Black Sea reflected in different studies provides an interesting picture of the evolution of human communities in this area. (Genov, 2016; Lericolais et al. 2009) (see Figure 2).

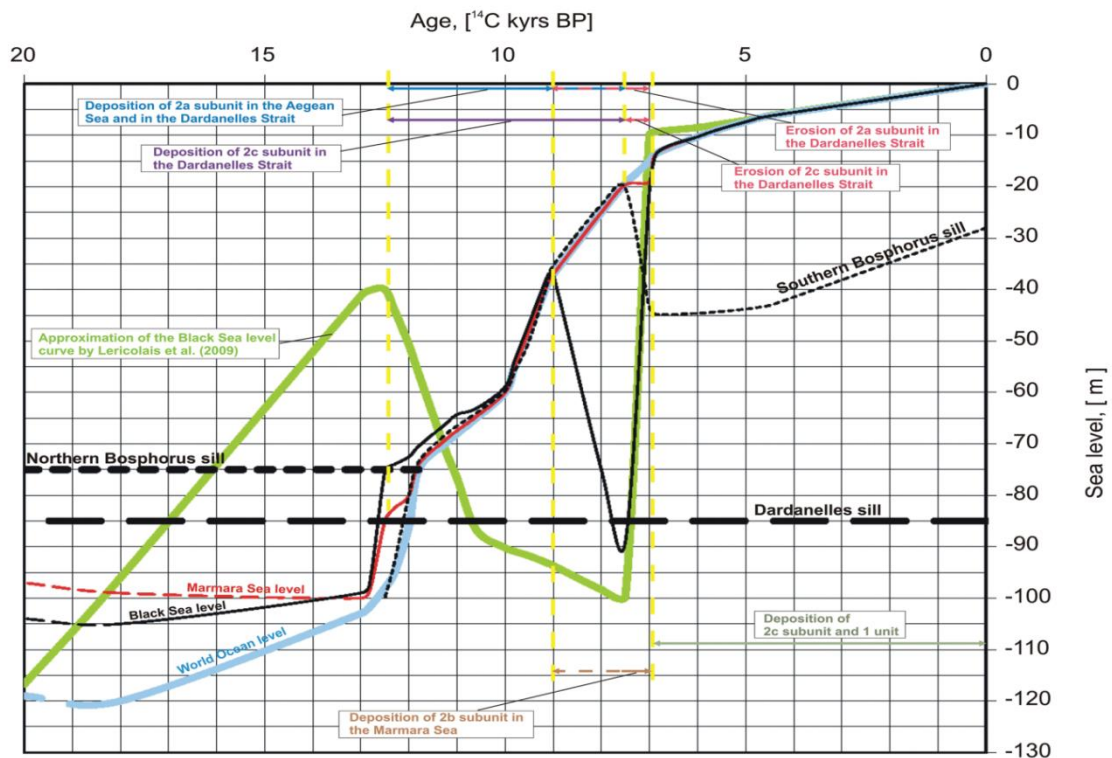


Figure 2. Comparison of the proposed Black Sea level evolution reflected in different studies and its correlation with that of Dardanelles Strait, the Aegean Sea and the Marmara Sea. (Genov, 2016)

The practice of classifying objects according to perceived similarities is the basis for much of science. Organizing data into sensible groupings is one of the most fundamental modes of understanding and learning. As an example, a common scheme of scientific classification puts organisms in to taxonomic ranks: domain, kingdom, phylum, class, etc.). Cluster analysis is the formal study of algorithms and methods for grouping objects according to measured or perceived intrinsic characteristics. Cluster analysis does not use category labels that tag objects with prior identifiers, i.e., class labels. The absence of category information distinguishes cluster analysis (unsupervised learning) from discriminant analysis (supervised learning). The objective of cluster analysis is to simply find a convenient and valid organization of the data, not to establish rules for separating future data into categories. (Jain, 2010)

Based on these studies, five stages of Danube Delta formation have been identified:

1. a slow rise in the Black Sea level (up to -40 - -35 m) in the Palaeolithic Period,
2. a very high drop of the Black Sea level (from -40 - -35 m to -100 - -90 m) in Early Mesolithic,
3. a catastrophic increase of the Black Sea level (from -100 - -90 m to -15 - -10 m) at the middle of the Mesolithic Period,
4. the stabilization of the Black Sea level (around -7 or -6 m) and the formation of the sea bays in the second half of the Mesolithic Period,
5. the main stage of development of the Danube Delta.

The cluster separation was performed based on data shown in Table 1, where logistic map classify the data by clusters.

Table 1. The cluster separation

Period	X_{n+1}	clusters
Paleolithic Period	0.062469	1
Early Mesolithic Period	0.019647	2
Late Mesolithic Period	0.046796	3
Neolithic Period	0.086191	4
Early Bronze Age	0.096037	5
Middle Bronze Age	0.100649	6
Iron Age	0.101172	6
Greek Period	0.100124	6
Dacian - Gaetic Period	0.097064	7
Roman Period	0.095091	7
Byzantine Period	0.079479	8

These data were ultimately correlated with those related to the classification of archaeological sites based on their spatial distribution. Dividing on clusters was performed on a scale of 0 to 8.

All data used in this study were georeferenced in the Stereo 70 System, on the Krasovsky 1940 ellipsoid, the Black Sea 1975 altimetric reference system.

RESULTS AND DISCUSSION

Within the Danube Delta, 8 clusters were identified based on the spatial distribution, marked with circles:

1. a first class of sites are those that could not be correlated, these would include the Bronze Period and the Byzantine Period in the Sulina Delta area (Cluster 0 – White colour);
2. a first cluster comprises the central part of the Sulina Delta and the Byzantine Period in the Sfântu Gheorghe Delta area (Cluster 1 – Red colour);
3. the second comprises the area between Dinogetia Promontory and Noviodunum Promontory (Cluster 2 – Orange colour);
4. the third is placed in the area of Lake Sinoe (Cluster 3 – Yellow colour);
5. the fourth comprises the area between Somova and Beştebe (Cluster 4 – Green colour);
6. the fifth comprises the area between Beştebe and Dunavăț (Cluster 5 – Blue colour);
7. the sixth cluster comprises the Babadag and Enisala area (Cluster 6 – Indigo colour);
8. cluster number seven comprises the Enisala - Dunavățul de Jos area (Cluster 7 – Violet colour);
9. cluster number eight comprises the area Noviodunum Promontory and Balta Somovei the western part (Cluster 8 – Magenta colour).

These, together with the values of correlation, played a feed-back role (see Figure 3).

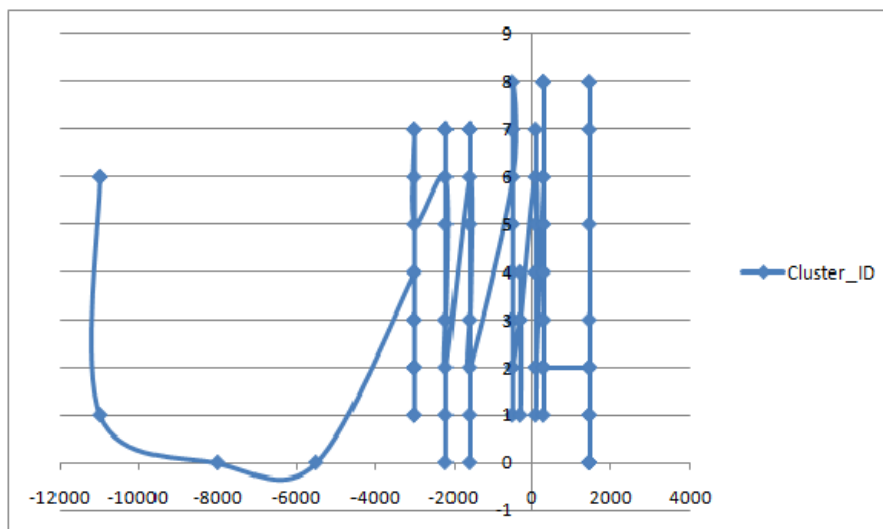


Figure 3. The spatial distribution

The data included in Cluster 0 refer to the evolution of the Chilia Delta, between the Chilia levee and the Letea sandbank and the regression of the Sulina Delta. This is also why the data could not be spatially correlated.

The evolution of the Lower Danube Sector according to the logistic map is marked by differently coloured triangles (see Figure 4 - 11).

Based on the connectivity-based clustering (hierarchical grouping), observations made within the archaeological sites were grouped into classes (groups or clusters) of similar elements (historical periods). (Panait et al., 2018)

However, existing theories and algorithms have difficulties to define and identify clusters in trees. (Yu et al., 2015)

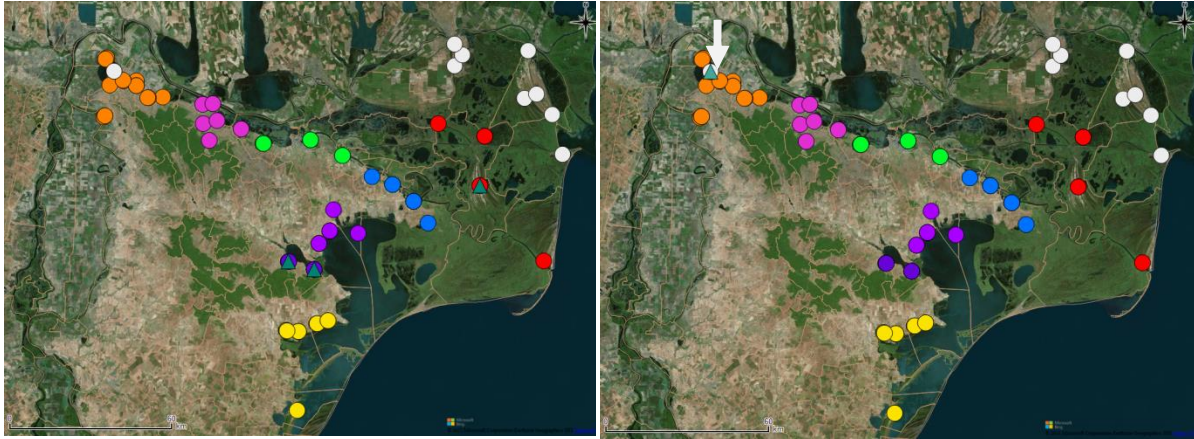
The presence of a hierarchy and the scale-free property impose strict restrictions on the number and the degree of cohesiveness of the different groups present in a network, which can be captured in a quantitative manner using a scaling law, describing the dependence of the clustering coefficient on the node degree. We use this scaling law to identify the presence of a hierarchical architecture in several real networks, and the absence of such hierarchy in geographically organized webs. (Erzsebet & Barabasi, 2003)

A method for the identification of the hierarchical organization of nodes in a network must fulfill two requirements: (i) It must be accurate for many types of networks, and (ii) it must identify the different levels in the hierarchy as well as the number of modules and their composition at each level. (Sales-Pardo, 2007)

The first attempts to reconstruct the ancient Histrian landscape were concentrated on the city and its connection with the land. Vasile Pârvan 4 suggested that apoikia (colonies) had been founded on an island, occupying a rocky promontory, which did not had a direct connection with neither the land nor the necropolis. (Preoteasa et al., 2012)

The earliest archaeological remains are from the Palaeolithic and Early Mesolithic. These were correlated very well with the Black Sea level since then. A slow rise in the Black Sea level (up to -40 - -35 m) in the Palaeolithic Period and a very high drop of the Black Sea level (from -40 - -35 m to -100 - -90 m) in Early Mesolithic had a similar impact on human settlements (see Figure 4). These are also reflected by the values of the clusters in Palaeolithic (0.062469) and Early Mesolithic (0.019647). These indicate a period when the Danube lower sector area had a different aspect (see Figure 5) than the

current one (Clusters 1 and 2). Of course, the values of the Black Sea level over the historical periods have been compared to the present one.



Figures 4 and 5. Black Sea lake stage (cluster 1 and 2)

The Black Sea level increase catastrophically (from -100 - -90 m to -15 – -10 m) at the middle of the Mesolithic Period (0.046796). That is indicated by the presence of human settlement in the Dinogetia Promontory area only (see Figure 6). Also, in the second half of the Mesolithic Period the stabilization of the Black Sea level (around -7 or -6 m) and the formation of the sea bays started (Cluster 3).

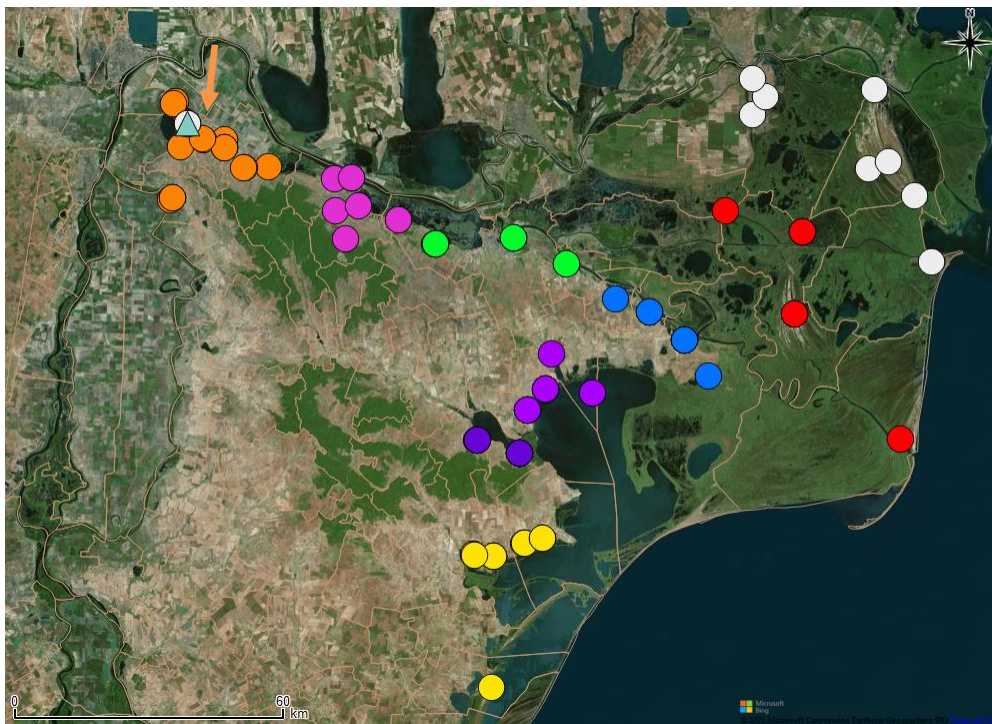


Figure 6. Marine transgression stage (the arrow indicates the limit)

Thus, there is a very big difference, at the Neolithic level, between the tendency to increase the Black Sea level and the spatial arrangement of the archaeological sites. This distribution of the sites indicate

a rapid transition from the lagoon phase to the delta itself (0.086191). Developing initially coastal belt followed shortly by an evolution of the Delta St. George I and Delta Sulina in the fluvial side (see Figure 7). In the north, the Delta of Chilia begins to develop. These observations are strengthened by placing, in terms of spatial distribution, age Neolithic archaeological sites in the central part of the delta (Mile 23 and on sandbar up to Crișan, on old Danube way) in Cluster 4.

Also in the Neolithic Period, small islands appeared in the area between Dinogetia and Aegysus Promontory.

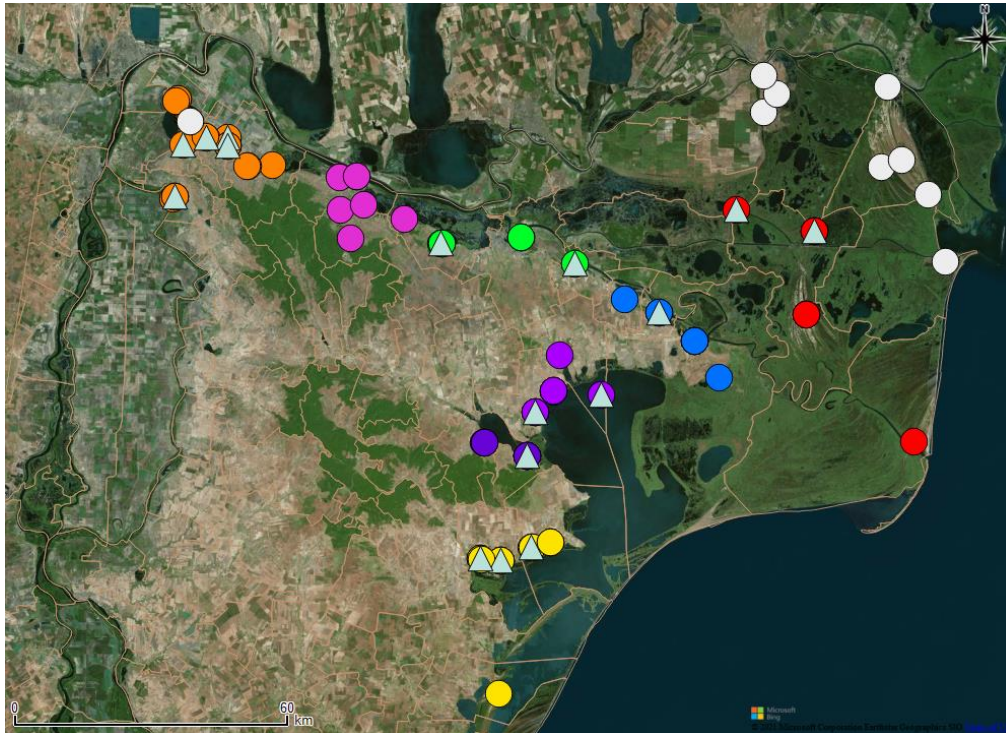


Figure 7. The lagoon phase of the Danube Delta Gheorghe I Delta and the Delta Sulina (fluvial side), Danube River follow the main land shape (Neolithic Period)

In the Early Bronze Age (cluster 5) we have an increase in the speed of evolution in the area of the lower sector of the Danube from 0.086191 to 0.096037. The highest areas are targeted during this period. This would indicate a normal reaction to the development of the Sulina Delta in its fluvial part. This evolution determined a population migration (see Figure 8).

At the same time, between the Dinogetia and Noviodunum promontories, the Danube follows the outline of the mainland. The junction with Danube with Siret and Prut rivers was further south than the current one.

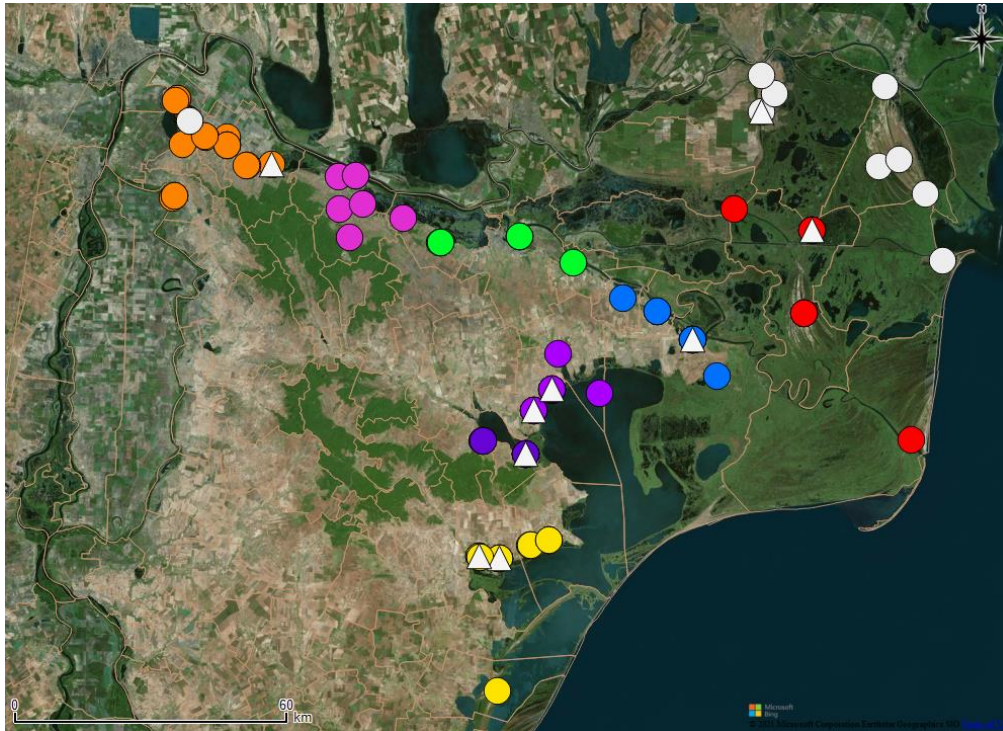


Figure 8. The phase of the Danube Delta Gheorghe I Delta and the Delta Sulina (fluvial side), Danube River follow the main land shape (Early Bronze Age)

In the 6th cluster we have sites from the Middle Bronze Age, the Iron Age and the Hellenistic Period (see Figure 9). The settlements are distributed along the old Danube way, the Sf. Gheorghe branch and Sulina branch.

The rate of evolution of the Danube Delta increases from the Middle Bronze Age to the Iron Age and begins to decline towards the Greek Period.

In the Iron Age, the Black Sea level continues to rise. Based on these data we can say that the delta evolved between the Danube Promontory and the southern end of the Initial Belt (along Sf. Gheorghe Arm), on the old route of the Sulina branch (passing the Initial Coastal Cord) and, in the north, the Chilia branch separates the Chilia promontory from the mainland (transforms the Chilia headland into a levee).

Between the promontories of Dinogetia-Noviodunum the Danube begins to evolve towards a route close to the present one. At the same time, the Siret and Prut rivers begin to create their own deltas. The reduction of the number of human settlements in the Iron Age betrayed this fact. In the Somova-Parcheş area, the Danube flows along two branches.

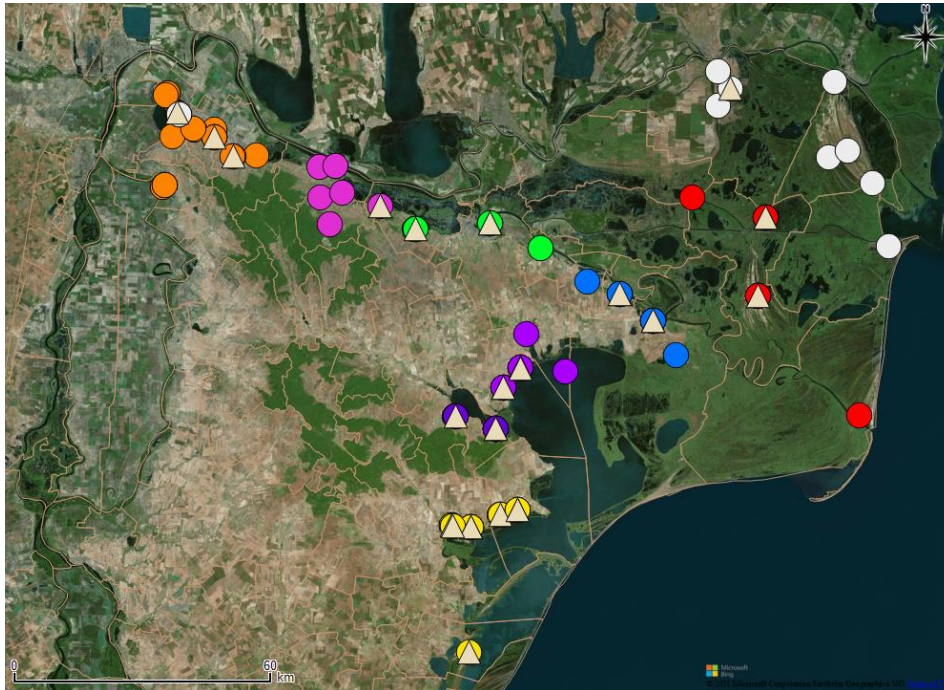


Figure 9. The sub-phase of the Delta (fluvial side) with Delta Sulina (fluvial side and northern half of marine side) and the Chilia Delta (fluvial side - half) stage (cluster 6)

The spatial placement of the Chilia beam area and the Letea sandbank in the same cluster demonstrates that the development of the Letea sandbank is linked to the evolution of the Chilia branch rather than to the Sulina branch. This fact is also confirmed by the presence of human settlements, from the Neolithic Period to the Early Roman Period, on a sandbar up to Crişan, on an old Danube path.

The presence of these settlements also demonstrates the fact that the Tulcea Gulf barrier was not achieved by a continuous Initial Belt (it was a succession of sandbanks). The northward crossing of the coastline by the Sulina branch does not affect the human settlements as far as the evolution of the Chilia branch is concerned.

The human settlements of the Getic Period correlate better than those of the Roman Period. This fact shows that the Danube is changing its course and flow in the Roman period. Also, in the Roman period, Sulina Delta reaches its maximum development, advancing largely towards the east and the Somova-Parceş area begins to close from the west (see Figure 10).

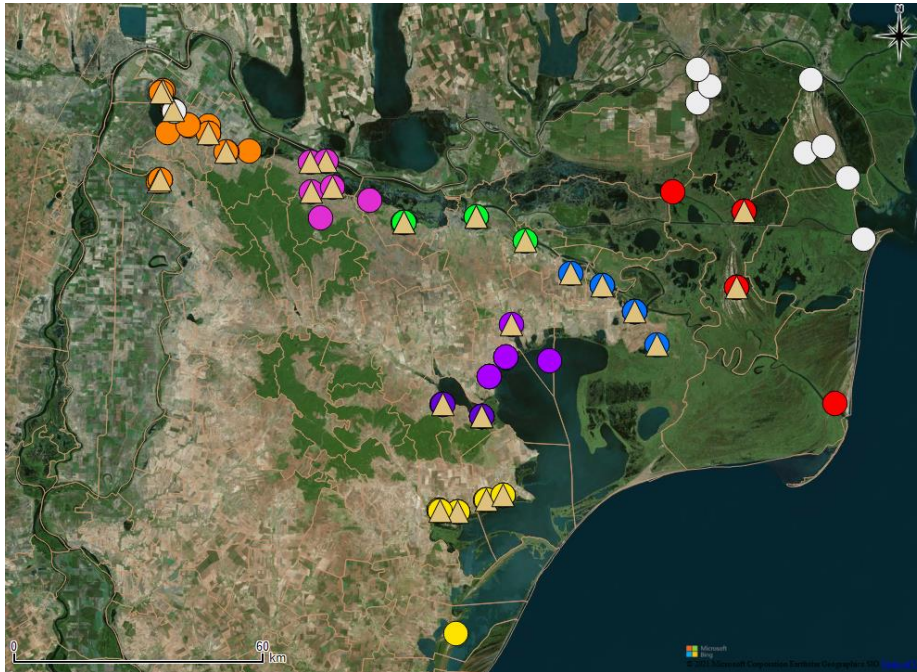


Figure 10. The phase of the Delta Sulina (marine side) and the Chilia Delta (fluvial side), between Dacian Period – Roman Period (cluster 7)

In the Byzantine period, the Danube is approaching the current way, and the Sulina Delta is eroded to a shape close to the current one (see Figure 11).

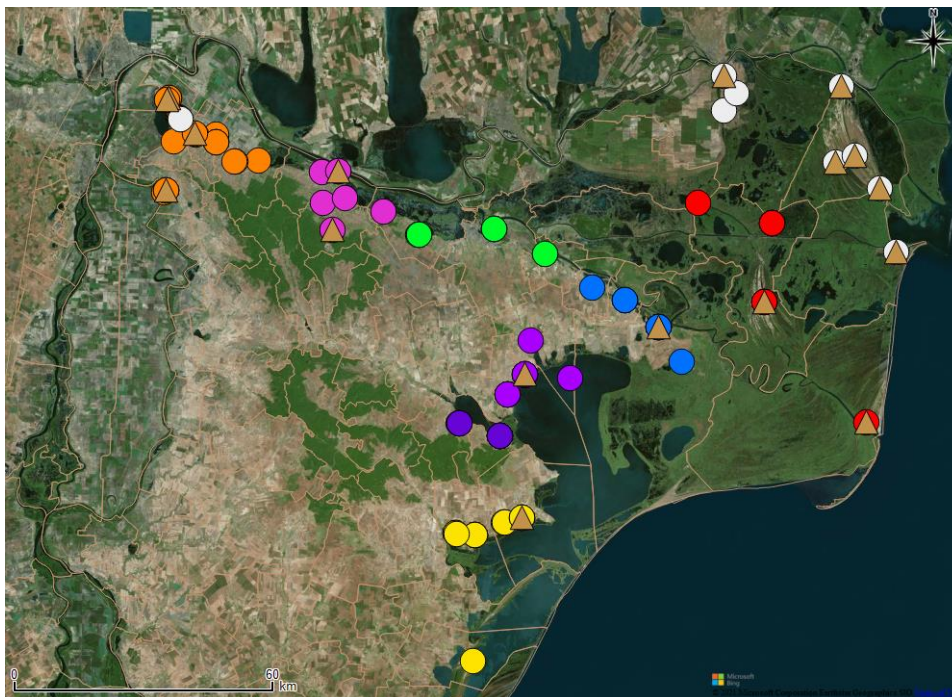


Figure 11. The phase of the Delta Sulina erosion and the development of the Lagunar Complex (Byzantine Period), the Danube River begins the development of the Măcin-Grindu sector and the Somova-Parches area (Byzantine Period)

Thus, the "behaviour" of the Danube, in the area of its lower sector, was analysed on the basis of the "behaviours" that could be directly observed. That is, the evolution of the Black Sea level and the distribution of human settlements.

CONCLUSIONS

The level of the Black Sea has varied over time and not increased steadily. Thus, when the sea level increased the Danube Delta evolved on the fluvio-maritime side, and when the sea level was reduced the development of the delta is made in its fluvial part.

It is interesting that the formation of the Letea sandbank is related to the evolution of the delta in the area of Lake Thiagola (Cluster 0). This evolution is placed between the Bonz and the Byzantine Period.

The values of the fractal dimensions indicate an increase of the evolution speed of the course of the Danube from the Neolithic Period to the Iron Age, in the area of its lower sector. Thus, the fractal size values are higher in the Dacian Period than in the Early Bonz, Roman and Byzantine Periods. This fact indicates that the speed of evolution of sedimentation processes in the lower Danube Sector was quite high in the Dacian Period.

Another important aspect is that the Caraorman sandbank is fully developed in the Roman period, while the evolution of the Letea sandbank extends to the Byzantine period. The heaviness of the human settlements in the Byzantine Period, in the area of the Letea sandbank and their reduction in the area of the Cotul Piscii, indicates that the evolution of the lower Danube sector is undergoing major transformations. Thus, the Letea sandbank is almost completely formed while the sector between Dinogetia and Aegyssus Promotory is evolving.

However, the presence of human settlements in the area of the sand bank Letea indicates the presence of three things: fresh water, wood (for construction and fire) and exit to the sea. So, it is very possible that the Letea and Caraorman sandbanks have been occupied by forests since the Getic Period (there are traces of Getic human settlements on the Caraorman sandbank). During this time, the sector between Dinogetia and Aegyssus Promotory is occupied by small islands. The level of the Danube rises from the Neolithic Period to the Byzantine period. A first equilibrium point being reached in the Getic Period.

But, in the Late Roman Period, the Danube level is experiencing a new growth, probably due to the evolution of agriculture, stimulated by Pax Romana. This increase in the level of the Danube resulted in the emergence of small islands between the Dinogetia and Aegyssus promontories.

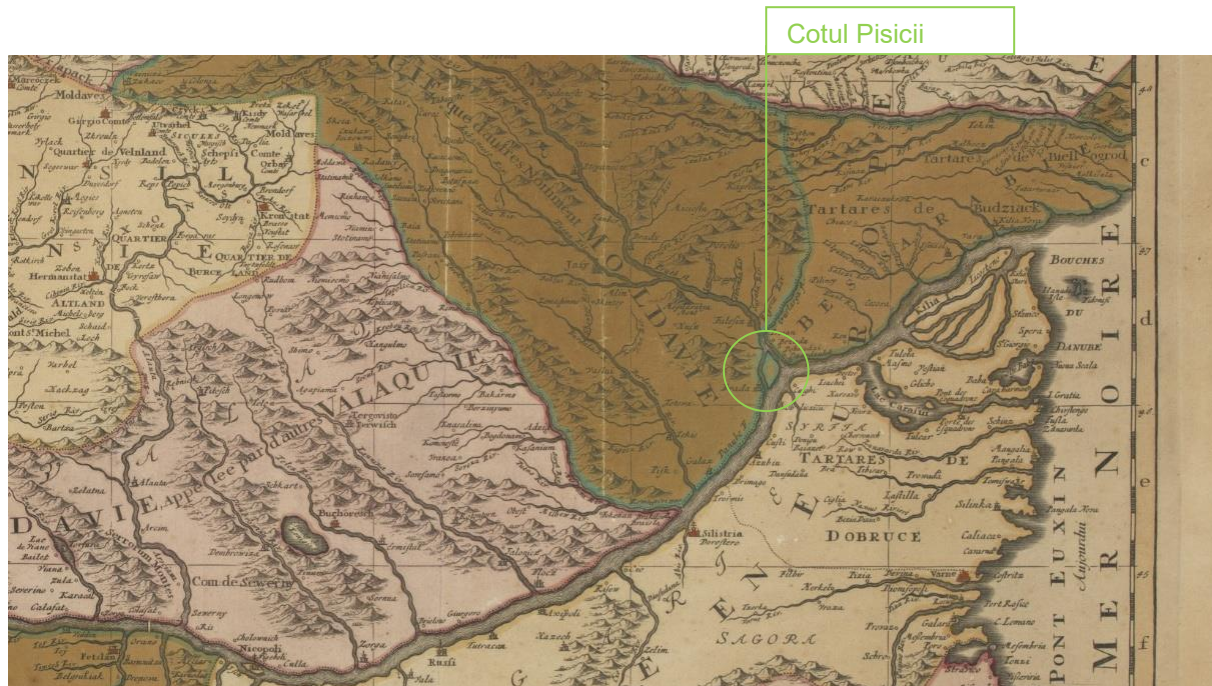


Figure 12 The Danube Delta map, published in Amsterdam, 1710 (COVENS J., MORTIER, C.)

Thus, although the variations of the Black Sea level did not directly influence the distribution of human settlements, however, the impact of these variations on the lower Danube course was reflected on them. So, "without being fully conscious of the influence" of the Black Sea level variation, the evolution of lower Danube sector can be described on the basis of the distribution of human settlements (see Figure 12).

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